

A Modified Smith Predictor with an Approximate Inverse of Dead Time

A compensator that approximates the inverse of dead time at low frequencies in a modified Smith predictor (MSP) control system is proposed, and the design of an MSP control system for it is given. The performance of the MSP with compensator, using a first-order element as a low-pass filter, is described. Analysis and simulation results show that the compensator improves performance in the disturbance rejection of the original Smith predictor.

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Introduction

The existence of time delay in a process transfer function causes major difficulties in the design and implementation of control. The dead-time compensation technique known as the Smith predictor (SP; Smith, 1957), which mitigates the problems of process time delay in control, has been the subject of numerous studies. Two major issues have been raised concerning the effectiveness of the SP. One is its sensitivity to modeling error; the other is its poor disturbance rejection capability. The studies of Francis (1980), Hocken et al. (1983), Horowitz (1983), Ioannides et al. (1979), Meyer et al. (1976), Palmor (1980), Palmor and Shinnar (1981), Palmor and Powers (1985), Romagnoli et al. (1988), and Laughlin et al. (1987) have addressed the issue of SP modeling error sensitivity. Attempts to improve the SP disturbance rejection capability have included the modified Smith predictor (MSP) of Watanabe et al. (1983), who proposed a modification that would reject input disturbance. Other authors, such as Wong and Seborg (1986), Wellons and Edgar (1985, 1987), and Doss and Moore (1982), have proposed using analytical predictors to reject output disturbances. Both approaches to the disturbance rejection problem tried to provide different compensators that implicitly approximate e^{-Ls} for disturbance rejection.

By use of the MSP, nominal performance in disturbance rejection can be improved significantly, but very little work has been addressed to incorporating robust stability in designing the compensator. As the system may be sensitive to high-frequency signals and modeling errors, ignorance of its robust stability becomes a major design limitation. Other limitations such as the requirement of a state-space formulation and tedious matrix algebraic equations are also found.

In this paper a predictive control for disturbance rejection within the framework of a general MSP is considered first. As an approximate inverse of dead time is needed in the MSP, a compensator with a general structure for such an approximation is proposed. The compensator has an important feature in that it serves as a predictor in the low-frequency range only and will not amplify the effect of signals at high frequencies. Design methodology for the MSP and the proposed compensator is also given. Compared with that of Watanabe et al., it is simple and transparent in design and in the incorporation of performance and robust stability issues.

General Modified Smith Predictor Control System

Consider the feedback control system shown in Figure 1. The output of the system, $y(s)$, can be written:

$$y(s) = G_{po}(s)e^{-Ls}u(s) + d(s) \quad (1)$$

where $G_{po}(s)$ is the process without delay, $u(s)$ is the control input, and $d(s)$ is the output disturbance. It should be noted that $d(s)$ can be either an output disturbance or the result of input disturbance.

Perfect predictive control requires that the future value of $y(s)$, with L units of time, be fed back to the controller, that is,

$$\begin{aligned} e^{Ls}y(s) &= e^{Ls}G_{po}(s)e^{-Ls}u(s) + e^{Ls}d(s) \\ &= G_{po}(s)u(s) + e^{Ls}d(s) \end{aligned} \quad (2)$$

Since a disturbance's future value can only be obtained by estimation, Eq. 2 is rewritten as:

$$y_p(s) = G_{po}(s)u(s) + P(s)\hat{d}(s) \quad (3)$$

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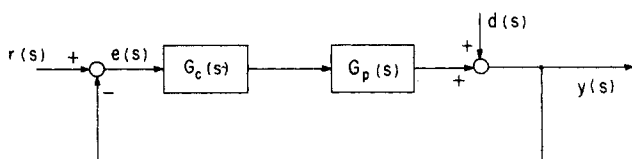


Figure 1. General feedback control system.

where $y_p(s)$ is the estimated value of $e^{Ls}y(s)$, and $P(s)$ is a predictor for the estimated disturbance $\hat{d}(s)$.

It is thus clear that $P(s)$ in Eq. 3 should aim at approximating e^{Ls} .

Now, in Eq. 3, $\hat{d}(s)$ can be estimated as

$$\begin{aligned}\hat{d}(s) &= y(s) - G_{po}(s)e^{-Ls}u(s) \\ &= y(s) - y_m(s)\end{aligned}\quad (4)$$

Then by inserting Eq. 4 into Eq. 3, the result is:

$$y_p(s) = G_{po}(s)u(s) + P(s)[y(s) - G_{po}(s)e^{-Ls}u(s)] \quad (5)$$

An illustration of the use of an Eq. 5 prediction in feedback control is given in Figure 2. Note that with $P(s) = 1$ the result is simply the Smith prediction control system. If, however, the system uses z-transform, and the compensator $P(z)$ is set equal to $A(z)$ (as in Wong and Seborg, 1986, or Wellons and Edgar, 1987), then the result is known as the analytical predictor or the generalized analytical predictor control system. With disturbance shifted from output to input, the system is almost identical to that of Watanabe et al. (1983). The control system resulting from the use of disturbance predictor $P(s)$ of Eq. 2 within the framework of the Smith predictor is what we here call a general modified smith prediction control system.

Design Considerations

According to Figure 2, when $G_p = \hat{G}_p$ the output $y(s)$ is:

$$y(s) = \frac{G_c G_p P}{1 + G_c G_{po}} r' + \frac{1 + G_c G_{po} - P G_c G_p}{1 + G_c G_{po}} d \quad (6)$$

where $r' = P^{-1}r$.

Let d_i and d_o represent the input and output disturbances, respectively, and d represent the total effect of the disturbances at the output,

$$d = G_p d_i + d_o \quad (7)$$

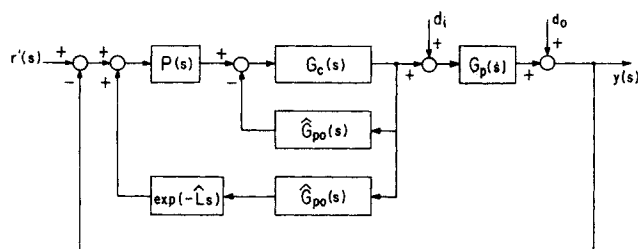


Figure 2. General modified Smith predictor control system.

Then we have

$$\begin{aligned}e(s) &= r(s) - y(s) \\ &= \frac{1 + G_c G_{po}(1 - e^{-Ls})}{1 + G_c G_{po}} r \\ &\quad - \frac{1 + G_c G_{po}(1 - P e^{-Ls})}{1 + G_c G_{po}} (G_p d_i + d_o)\end{aligned}\quad (8)$$

If the dead time is eliminated from the closed-loop characteristic equation, then G_c can be designed in such a way that for most frequencies of concern we have:

$$\frac{1}{1 + G_c G_{po}} \approx 0 \quad (9)$$

and

$$\frac{G_c G_{po}}{1 + G_c G_{po}} \approx 1 \quad (10)$$

From Eq. 8, the system error becomes:

$$e(s) \approx (1 - e^{-Ls})r - (1 - P e^{-Ls})(G_p d_i + d_o) \quad (11)$$

In the case of input disturbance ($d_o = 0$), Watanabe et al. used P to cancel the undesired poles of G_p with the zeros of $(1 - P e^{-Ls})G_{po}$.

However, it is better that P be designed to approximate the inverse of e^{-Ls} so that

$$1 - P e^{-Ls} \approx 0 \quad (12)$$

for most frequencies of concern. In the next section we provide a general structure for $P(s)$ which serves as an approximation to the inverse of dead time, e^{Ls} .

Now, with a modified compensator $P(s)$ as the approximation to the inverse of dead time, the system can be analyzed as follows.

From Figure 2, the equivalent feedback controller $G_c^*(s)$ and the return difference $R_d(s)$ of the system can be derived:

$$G_c^*(s) = \frac{P(s)G_c(s)}{1 + G_c(s)\hat{G}_{po}(s) - P(s)G_c(s)\hat{G}_p(s)} \quad (13)$$

and

$$\begin{aligned}R_d(s) &= 1 + G_c^*(s)G_p(s) \\ &= \frac{1 + G_c(s)\hat{G}_{po}(s) - P(s)G_c(s)\hat{G}_p(s) + P(s)G_c(s)G_p(s)}{1 + G_c(s)\hat{G}_{po}(s) - P(s)G_c(s)\hat{G}_p(s)}\end{aligned}\quad (14)$$

Let $Q(s)$ be the complementary sensitivity function of a delay-free system:

$$Q(s) = \frac{G_c(s)\hat{G}_{po}(s)}{1 + G_c(s)\hat{G}_{po}(s)} \quad (15)$$

and define the multiplicative modeling error $I(s)$ as:

$$I(s) = \frac{\hat{G}_p - G_p}{\hat{G}_p} = 1 - \frac{G_{po}(s)}{\hat{G}_{po}(s)} e^{-(L-\hat{L})s} \quad (16)$$

then Eq. 14 can be written as:

$$R_d(s) = \frac{1 - P(s)Q(s)I(s)e^{-\hat{L}s}}{1 - P(s)Q(s)e^{-\hat{L}s}} \quad (17)$$

Therefore, the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$ of the system in Figure 2 are:

$$S(s) = \frac{1}{1 + G_c^*(s)G_p(s)} = \frac{1 - P(s)Q(s)e^{-\hat{L}s}}{1 - P(s)Q(s)I(s)e^{-\hat{L}s}} \quad (18)$$

and

$$T(s) = \frac{P(s)Q(s)(1 - I(s))e^{-\hat{L}s}}{1 - P(s)Q(s)I(s)e^{-\hat{L}s}} \quad (19)$$

When there is no modeling error (when $I(s) = 0$), Eqs. 18 and 19 become:

$$S(s) = 1 - P(s)Q(s)e^{-\hat{L}s} \quad (20)$$

and

$$T(s) = P(s)Q(s)e^{-\hat{L}s} \quad (21)$$

It is obvious that nominal stability requires that both $P(s)$ and $Q(s)$ be stable. When modeling error exists, apart from $P(s)$ and $Q(s)$ stability, it is also necessary that:

$$|P(j\omega)| < \frac{1}{|Q(j\omega)\hat{I}(\omega)|} \quad \forall \omega \quad (22)$$

here, $\hat{I}(\omega)$ denotes the maximum possible value of $|I(j\omega)|$ at each frequency.

On the other hand, the nominal performance of the system based on optimal H_2 design requires that $S(j\omega)W(j\omega)$ be minimized (Rivera and Morari, 1987). It is well known that:

$$\begin{aligned} \min_{|P|} \int_0^\infty |S(j\omega)|^2 d\omega \\ \rightarrow \min_{|P|} \int_0^\infty \{1 - P(j\omega)Q(j\omega)e^{-j\omega\hat{L}}\} \\ \cdot \{1 - P(-j\omega)Q(-j\omega)e^{j\omega\hat{L}}\} d\omega \quad (23) \end{aligned}$$

For nominal performance, it is desirable that the magnitude of $P(s)Q(s)$ in the frequency domain equals the value one for

each ω . However, this is not practical because noise, which often occurs in the high-frequency range, can jeopardize the performance of the system. Therefore, we can require that:

$$|P(j\omega)Q(j\omega)| \rightarrow 1 \quad \text{as } \omega < \omega_c \quad (24)$$

Where ω_c is the upper frequency of major concern regarding the inputs.

Thus, the method for designing the general MSP control system consists of two steps. First, a Smith predictor control system is designed according to existing methods (Laughlin et al., 1987). Second, $P(s)$ satisfying both Eqs. 22 and 24 is introduced. If robust performance is required, the conditions reported by Morari and Zafiriou (1989) should be considered.

The Compensator $P(s)$ for Approximating $\exp(+Ls)$

The dynamic compensator $P(s)$ in a modified Smith predictor control system attempts to approximate e^{Ls} for disturbance compensation. The design of $P(s)$ in other systems is based on dynamic models of disturbances. But such models are not easy to obtain and are usually accompanied by modeling errors, especially when the disturbance has time-varying characteristics.

Consider $P(s)$ being given as follows:

$$P(s) = \frac{1 + B(s)}{1 + B(s)e^{-Ls}} \quad (25)$$

Figure 3 shows the block diagram that implements the transfer function in Eq. 25. The following properties allow $P(s)$ to be an approximation to e^{Ls} in the low-frequency range:

1. If $B(s)$ is a low-pass element with high gain, then $P(s) \approx e^{Ls}$ in the low-frequency range.
2. If $B(s)$ is a low-pass element, then $P(s) \approx 1$ in the high-frequency range.

The proofs of properties 1 and 2 are straightforward. It is then clear that $P(s)$ will serve as a predictor for low-frequency signals but not, however, for those signals of high-frequency range. Furthermore, if $P(s)$ is given as a lead/lag element, as in the example of Watanabe et al. (1983), the system may be endangered by an amplification of the high-frequency portion of the disturbance.

A first-order lag is the most convenient choice for the low-pass component of Eq. 25:

$$B(s) = \frac{k}{\tau s + 1} \quad (26)$$

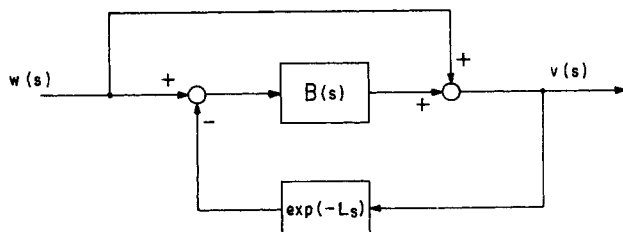


Figure 3. Block diagram for compensator $P^*(s)$ defined in Eq. 25.

We will designate $P^*(s)$ as the $P(s)$ that uses Eq. 26 for $B(s)$ in Eq. 25, that is

$$P^*(s) = \frac{1 + \frac{k}{\tau s + 1}}{1 + \frac{k}{\tau s + 1} e^{-Ls}} \quad (25a)$$

The compensator $P(s)$ proposed by Watanabe et al. (1983) must satisfy the following conditions in order to reject step disturbance at process input at satisfactory speeds.

1. $G_c(s)$ has an integrator
2. The poles of $P(s)$ can take arbitrarily preassigned values
3. The poles of $\hat{G}_{po}(s)$ in $\hat{G}_{po}(s) - P(s)\hat{G}_p(s)$ are canceled with its zero:

$$(s - \zeta_i)\{\hat{G}_{po}(s) - P(s)\hat{G}_p(s)\}_{s=\zeta_i} = 0 \quad (27)$$

$$\lim_{s \rightarrow 0} \{\hat{G}_{po}(s) - P(s)\hat{G}_p(s)\} = 0 \quad (28)$$

Using the $P^*(s)$ of Eq. 25a, the modified Smith predictor control system has the asymptotical properties of Eqs. 27 and 28, which effectively reject input disturbance. The following theorem will depict these asymptotic properties:

Theorem. A modified Smith predictor control system with a dynamic compensator $P^*(s)$ of Eq. 25a has the following properties:

1. The largest pole of stable $P^*(s)$ given in Eq. 25a can be arbitrarily assigned.
2. If $\hat{G}_{po}(s)$ has a pole that is close to the origin, Eq. 27 is satisfied asymptotically as the gain of the low-pass element becomes large.
3. If $\hat{G}_{po}(s)$ does not have a zero pole, then Eq. 28 holds. And if $\hat{G}_{po}(s)$ has a pole that is close to the origin, Eq. 28 is satisfied asymptotically as the gain of the low-pass element becomes large.

Proof of property 1. From Eq. 25a, one has the pole of $P^*(s)$ at

$$s = -\frac{1 + ke^{-Ls}}{\tau} \quad (29)$$

Although Eq. 29 may have many roots, by varying k and τ for a given L the largest root can be arbitrarily relocated under the requirement that $P^*(s)$ must be stable.

Proof of property 2. Equation 27 implies that $1 - P^*(s)e^{-Ls}$ should have zeros at poles of $\hat{G}_{po}(s)$. That is:

$$1 - P^*(s)e^{-Ls} = 0 \quad \text{at } s = \zeta_i$$

where ζ_i is any pole of $\hat{G}_{po}(s)$ close to the origin.

From Eq. 25a, we have:

$$1 - P^*(s)e^{-Ls} = \frac{(1 - e^{-Ls})(1 + \tau s)}{\tau s + 1 + ke^{-Ls}} \quad (30)$$

As $k \rightarrow \infty$, $1 - P^*(s)e^{-Ls} \rightarrow 0$ at any value of s .

Proof of property 3.

1. As $\hat{G}_{po}(0)$ is finite and $P^*(0) = 1$, it is obvious that Eq. 28 holds at $s = 0$.

2. The asymptotical property of Eq. 28 implies:

$$\frac{d\{1 - P^*(s)e^{-Ls}\}}{ds} = 0 \quad \text{at } s = 0$$

Equations 25 and 26 lead to:

$$1 - P^*(s)e^{-Ls} = \left[L - \frac{kL}{(1+k)} \right] s + \frac{[L^2 - 2TkL + 3kL^2]}{2(1+k)^2} s^2 + \dots \quad (31)$$

then,

$$\frac{d\{1 - P^*(s)e^{-Ls}\}}{ds} = L - \frac{kL}{1+k} \quad \text{at } s = 0 \quad (32)$$

which approaches to zero as k goes to infinity.

Thus, the dynamic compensator $P^*(s)$ given in Eq. 25a has the same asymptotic properties and the same ability to reject input disturbance as the compensator of Watanabe et al. (1983). It has, however, the advantages over Watanabe's lead/lag compensator of having fewer design constraints when faced with high-frequency modeling error and sensor noise. Besides, when model mismatches exist, one can still assure the robustness of $P^*(s)$ by considering that $P^*(s)$ is constrained by the inequality in Eq. 27.

Design Procedures for $P(s)$

In order to guarantee the stability of the system under possible modeling errors, $P(s)$ must follow Eq. 22. In other words, the curve of the inverse of $|Q(j\omega)\hat{I}(\omega)|$ represents a necessary constraint for $|P(j\omega)|$. The magnitude curves of all $P(s)$ designs should lie beneath this constraint. Furthermore, where $P(s)$ follows Eq. 24, system performance will improve. Thus, the magnitude curve of $P(s)$ should be close to the curve of $1/|Q(j\omega)|$ over the frequency range of concern, since noise that affects the system usually occurs at high frequencies. As a consequence, in the Smith predictor control system, $|Q(j\omega)|$ should roll off in the high-frequency range. Meanwhile $|Q(j\omega)|$ should not be altered at high frequency when $P(s)$ is introduced. This is to say, $|P(j\omega)|$ should remain as close to unity as possible in the high-frequency range. Thus the design of $P(s)$ is simply a matter of choosing a proper $P(s)$ such that $|P(j\omega)|$ lies as close as possible to the curve of $1/|Q(j\omega)|$ and beneath the curve of $1/|Q(j\omega)\hat{I}(\omega)|$.

We will elucidate the design of $P(s)$ by using $P^*(s)$ as an example in the following.

Since stability is of primary concern in designing the compensator $P^*(s)$, we must find the maximum value of k for which $P^*(s)$ is stable for any given τ and L in Eq. 25.

Since,

$$k_{\max} = \sqrt{1 + \omega^2 \tau^2} \quad (33a)$$

where ω^* satisfies:

$$\tan^{-1}(\omega^* \tau) + (\omega^* \tau) \left(\frac{L}{\tau} \right) = \pi \quad (33b)$$

Eq. 25 gives the value of $P^*(s)$ at ω^* as:

$$|P^*(j\omega^*)| = \frac{\sqrt{(1 + k_{\max})^2 + \tau^2 \omega^{*2}}}{\sqrt{\left\{ 1 + k_{\max} \cos \left[(\omega^* \tau) \left(\frac{L}{\tau} \right) \right]^2 + \left\{ \omega^* \tau - k_{\max} \sin \left[(\omega^* \tau) \left(\frac{L}{\tau} \right) \right] \right\}^2}} \quad (34)$$

From Eqs. 33a, 33b, and 34, it is then concluded that

$$k_{\max} = f_1(L/\tau) \quad (35)$$

and

$$|P^*(j\omega^*)| = f_2(L/\tau) \quad (36)$$

The value of L in $P^*(s)$ is assumed normally to be the nominal time delay in the process model. However, when the process time constant is large, it is found that, in matching the constraint of Eq. 24, one may also adjust L to improve the performance further. The resulting difference between the L used in the compensator and that of the process model may be considered as a virtual time delay caused by the large time constant, which needs to be compensated by means of the predictive compensator.

The design procedures of $P^*(s)$ for a modified Smith predictor control system can then be summarized as follows:

1. Design a feedback controller for a robust Smith predictor control system to meet performance and stability requirements.
2. Plot the curves, in the frequency domain, of the reciprocal of $|Q(j\omega)\hat{I}(\omega)|$ and of $|Q(j\omega)|$.
3. Choose a proper value of k , L/τ , and L under the constraints of Eqs. 35 and 36 so that the magnitude spectrum of the resulting $P^*(s)$ best fits the area beneath the curves in step 2. It should be noted that the parameter L in $P^*(s)$ does not necessarily equal the nominal dead time of the process.

Examples

1. Response to output disturbance

Consider a process given by Eq. 37:

$$G_p(s) = \frac{e^{-3s}}{10s + 1} \quad (37)$$

The model used in the Smith predictor control system is identical that of the process in Eq. 37.

In this example, the tuning parameters in G_c are given as $K_c = 1$, and $\tau_R = 10$. Figure 4 shows the constraints for both of nominal performance ($1/|Q(j\omega)|$) and robust stability ($1/|Q(j\omega)\hat{I}(\omega)|$). For different $B(s)$, the fit of $P^*(s)$ magnitude curves relative to the area beneath constraints is thus illustrated.

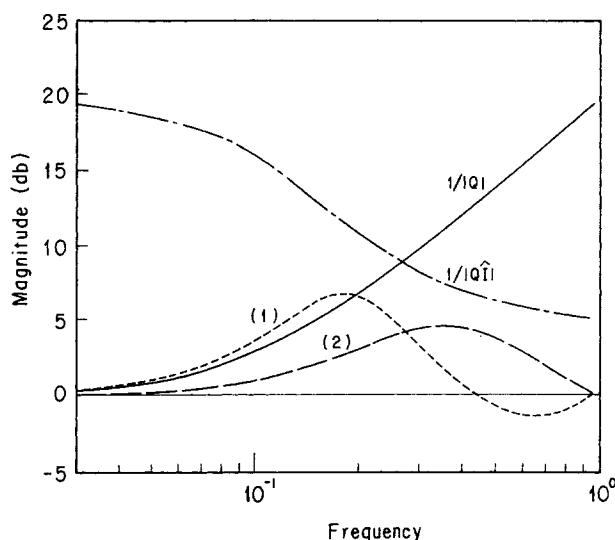


Figure 4. Frequency diagram of examples 1 and 2 for design of $P^*(s)$.

- (1) $B(s) = 6/(60s + 1)$, $L = 7$
(2) $B(s) = 10/(60s + 1)$, $L = 3$

The responses of such modified Smith predictor control systems to an output disturbance in Eq. 38 are given in the Figure 5.

$$d(s) = \frac{0.2}{s + 0.2} \frac{1}{s} \quad (38)$$

The responses in Figure 5 clearly show the effect of the tuning constants in $B(s)$. The results do reflect the expected asymptotic properties of the modified system that rejection of disturbance can be made faster by appropriate choice of the tuning constant k .

2. Response to input disturbance

Consider the same $G_p(s)$ as that in example 1 and a step load introduced at the input. The parameters of G_c are $K_c = 1$, $\tau_R =$

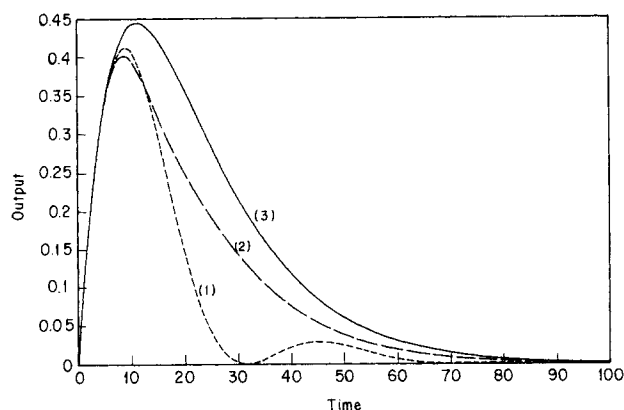


Figure 5. Responses of example 1 to output disturbance with different tuning parameters in $P^*(s)$.

- Output disturbance: $d(s) = 1/[s(12s + 1)]$
(1) $B(s) = 6/(60s + 1)$, $L = 7$
(2) $B(s) = 10/(60s + 1)$, $L = 3$
(3) Smith predictor

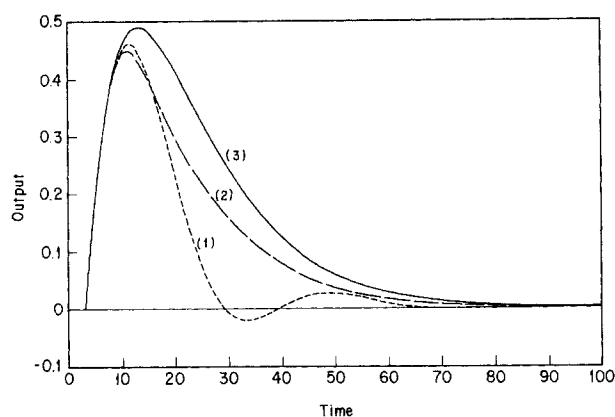


Figure 6. Responses of example 2 to unit step input disturbance with different tuning parameters in $P^*(s)$.

- (1) $B(s) = 6/(60s + 1)$, $L = 7$
- (2) $B(s) = 10/(60s + 1)$, $L = 3$
- (3) Smith predictor

10. The time domain responses of the system are given in Figure 6.

3. Effects of modeling error on $P^*(s)$ and a lead/lag element

This example shows a comparison between the modified system using the compensator $P^*(s)$ and the one with compensator in lead/lag form. Consider the same process model as that of example 1. The actual process is given by:

$$G_p(s) = \frac{0.95e^{-3s}}{10s + 1} + 0.1e^{-6s} \quad (40)$$

and the worst possible bound for stability is given by $1/|Q(j\omega)\hat{I}(\omega)|$ as in example 1.

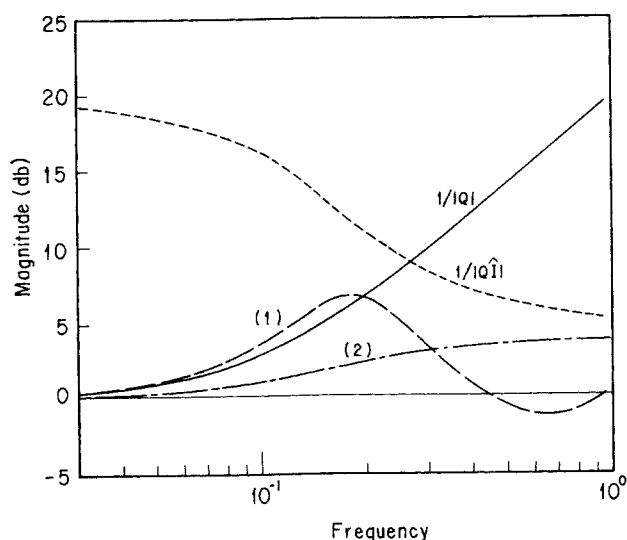


Figure 7. Frequency diagram of example 3 for design of $P^*(s)$ and a lead/lag compensator.

- (1) $B(s) = 6/(60s + 1)$, $L = 7$
- (2) $P(s) = (8s + 1)/(5s + 1)$

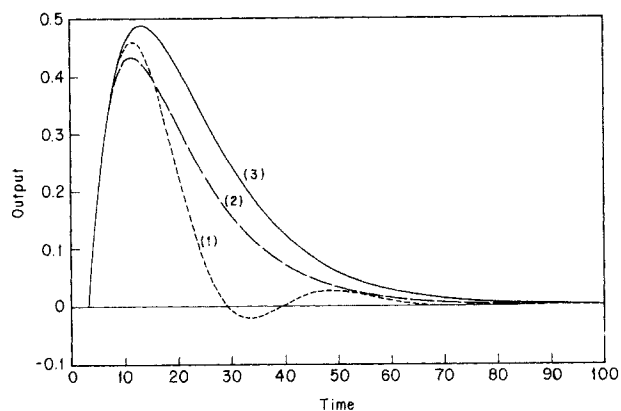


Figure 8. Responses of example 3 to unit step input disturbance, $P^*(s)$ vs. lead/lag element.

- (1) $B(s) = 6/(60s + 1)$, $L = 7$
- (2) $P(s) = (8s + 1)/(5s + 1)$
- (3) Smith predictor

Now, consider the compensators $P(s)$'s, both $P^*(s)$ and the lead/lag form $(t_1s + 1)/(t_2s + 1)$. The frequency domain diagram for designing compensators is shown in Figure 7. Because of the modeling error, the magnitude of lead/lag element at the high-frequency range is constrained. Tuning parameters that meet the robust condition are $t_1 = 8$ and $t_2 = 5$. The nominal responses of the systems to a unit input disturbance are shown in Figure 8. The responses when modeling error exists are shown in Figure 9.

Conclusions

Following the argument of this paper, we see that a modified Smith predictor requires a compensator that serves as an approximation to e^{Ls} for disturbance rejection. A general form of such a compensator is therefore proposed. The important features of the proposed compensator are:

1. It provides a general structure for approximating e^{Ls} .
2. It approximates e^{Ls} only at the low frequency range, but approaches a value of 1 as frequency increases. This property improves system performance without jeopardizing stability.

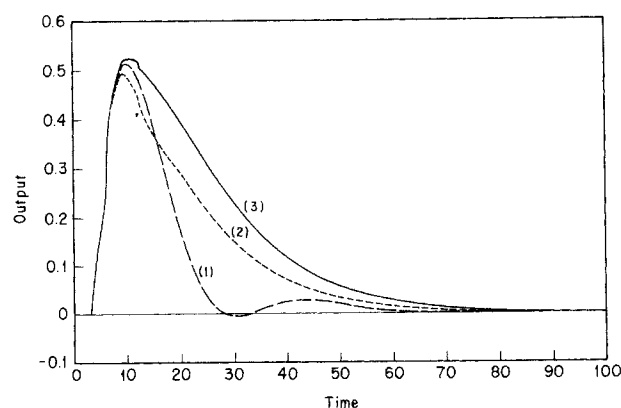


Figure 9. Responses of example 3 to unit step input disturbance with modeling error.

- (1) $B(s) = 6/(60s + 1)$, $L = 7$
- (2) $P(s) = (8s + 1)/(5s + 1)$
- (3) Smith predictor

3. It is good for rejecting both input and output disturbance.
4. It requires no prior knowledge about disturbance characteristics.

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Notation

$A(z)$ = analytical predictor
 $B(s)$ = a low-pass component in $P^*(s)$
 $d(s)$ = output disturbance
 $G_c(s)$ = equivalent feedback controller
 $G_p(s)$ = process transfer function
 $\hat{G}_p(s)$ = process model
 $G_{pd}(s)$ = free delay transfer function of the plant
 $\hat{G}_{pd}(s)$ = free delay transfer function of the model
 $I(s)$ = multiplicative modeling error, Eq. 16
 $\hat{I}(\omega)$ = maximum value of $I(j\omega)$ for each frequency
 L = process time delay
 $P(s)$ = predictor
 $P^*(s)$ = proposed predictor, Eq. 25
 $Q(s)$ = rational function
 $R_d(s)$ = return difference of the system
 $S(s)$ = sensitivity function
 $T(s)$ = complementary sensitivity function
 $u(s)$ = control variable
 $y(s)$ = output
 $y_p(s)$ = predictive output

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